Topics on Free Boundary Problems for Ideal Fluids (Mathematical Analysis in Fluid and Gas Dynamics)

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This article reviews the free boundary problems for the motion of an incompressible ideal fluid. These problems are typically classified into the following three types according to the geometrical cofigulations of fluid domain:
[I] Vortical water waves, i.e., fluid motion in a domain of infinite extent bounded by the upper free surface and the lower bottom of finite or infinite depth (in this case a dominant external force is due to a gravitation downward vertically),
[II] Circulating fluid around a celestial body, i.e., fluid motion around a rigid body with a compact free surface (in this case a dominant external force is due to a gravitation of the celestial body),
[III] Gaseous stars, i.e., fluid motion in a domain bounded by the free surface (in this case a dominant external force is due to a self-gravitational forcee).

In general, as the vector fields of external forces we should take not only the potential but the general form. Besides the forces mentioned above the effect of the surface tension is taken into account.

We have a long history and a lot of works discussing on these problems, however concerning the most fundamental study of the wellposedness of these problems there are not so many works. In this article we are inerested in just this wellposedness.

We begin with a classical description of the problem.

At time $t>0$ let $\Omega(t)$ be a domain in $\mathbb{R}^N (N=2,3)$ occupied by the fluid, which is bounded by a bottom Γ_b and a free surface $\Gamma_s(t)$:

$$\begin{aligned}\Gamma_b &= \left\{\mathbf{x}\in\mathbb{R}^N \mid F_b(\mathbf{x})=0\right\},\\ \Gamma_s(t) &= \left\{\mathbf{x}\in\mathbb{R}^N \mid F_s(\mathbf{x},t)=0\right\}.\end{aligned}$$

We always assume that $\Gamma_b\cap\Gamma_s(t)=\emptyset$ for any $t\geq 0$. Corresponding to problems [I], [II] and [III], we usually consider the fluid motion in the domains given by

$$F_b(\mathbf{x})\equiv x_N-b(\mathbf{x}'),\quad F_s(\mathbf{x},t)\equiv x_N-\eta(\mathbf{x}',t)\quad (\mathbf{x}=(\mathbf{x}',x_N)),$$

$$F_b(\mathbf{x})\equiv|\mathbf{x}|-b(\omega),\quad F_s(\mathbf{x},t)\equiv|\mathbf{x}|-\eta(\omega,t)\quad (\omega\in S^{N-1}),$$

$$\Gamma_b=\emptyset,\quad F_s(\mathbf{x},t)\equiv|\mathbf{x}|-\eta(\omega,t)\quad (\omega\in S^{N-1}),$$

respectively, where S^{N-1} is an (N-1)-dimensional sphere.

The motion of an incompressible ideal fluid is described in the Eulerian coordinates by

$$\varrho\frac{\mathrm{D}}{\mathrm{D}t}\mathbf{v}+\nabla_x p=\varrho\mathbf{f},\quad \mathrm{div}_x\mathbf{v}=0\quad\text{in}\quad \Omega(t),\ \ t>0, \tag{1}$$

where $\mathbf{v}=\mathbf{v}(\mathbf{x},t)$ is the velocity vector field, $p=p(\mathbf{x},t)$ is the pressure, $\mathbf{f}=\mathbf{f}(\mathbf{x},t)$ is a vector field of exterior mass forces, ϱ is the constant density of the fluid and $\mathrm{D}/\mathrm{D}t=\partial/\partial t+\mathbf{v}\cdot\nabla_x$ is a material derivative.

Note that correspondent to problems [I], [II] and [III], the suitable forms of the external forces are considered as

$$\begin{aligned}
&\mathbf{f}(\mathbf{x},t)=-\varrho g\nabla\Phi(\mathbf{x},t),\quad \Phi(\mathbf{x},t)=x_\perp,\\
&\mathbf{f}(\mathbf{x},t)=\varrho Mg\nabla\Phi(\mathbf{x},t),\quad \Phi(\mathbf{x},t)=\frac{1}{|\mathbf{x}|},\\
&\mathbf{f}(\mathbf{x},t)=4\pi\varrho g\nabla\Phi(\mathbf{x},t),\quad \Phi(\mathbf{x},t)=\int_{\Omega(t)}\frac{\varrho}{|\mathbf{x}-\mathbf{y}|}\,\mathrm{d}y,
\end{aligned}$$

where g is a gravitational constant, $x_\perp$ is a perpendicular component of $\mathbf{x}$ and M is a mass of a celestial body. In problem [II] we take the barycenter of the celestial body as an origin of the coordinate system.

The boundary conditions on $\Gamma_s(t)$ are

$$\begin{cases}\dfrac{\mathrm{D}}{\mathrm{D}t}F=0 & \text{(kinematic condition)},\\ p=p_e+2\sigma H & \text{(dynamic condition)},\end{cases} \tag{2}$$

and on Γ_b

$$(\mathbf{v}-\mathbf{v}_b)\cdot\mathbf{n}_b=0. \tag{3}$$

Here p_e is an atmosphere pressure, $\sigma(\geq 0)$ a coefficient of surface tension, H a mean curvature ($H>0$ if Γ_t is convex outside the fluid region), $\mathbf{v}_b$ velocity of Γ_b and $\mathbf{n}_b$ a normal vector to Γ_b.

Initial conditions are

$$\begin{cases}\Omega(0)=\Omega\quad (\Gamma_s(0)=\Gamma_s),\\ \mathbf{v}|_{t=0}=\mathbf{v}_0(\mathbf{x}),\quad \mathrm{div}\mathbf{v}_0=0,\quad \mathbf{x}\in\Omega.\end{cases} \tag{4}$$

Our aim is to find a solution $(\Omega(t),\mathbf{v}(\mathbf{x},t),p(\mathbf{x},t))$ for $t>0$ to problem (1) – (4).

It is convenient to write the problem in the Lagrangean coordinates ξ:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}=\mathbf{v}(\mathbf{x},t),\quad \mathbf{x}|_{t=0}=\xi, \tag{5}$$

which can be solved by the formula

$$\mathbf{x}=\xi+\int_0^t\mathbf{u}(\xi,\tau)\,\mathrm{d}\tau,\quad \mathbf{u}(\xi,t)=\mathbf{v}(\mathbf{x}(\xi,t),t). \tag{5$'$}$$

Let $\mathcal{G}(\xi,t) = G(\mathbf{x}(\xi,t),t)$. Then from (5) it follows that

$$\frac{\partial}{\partial t}\mathcal{G} = \frac{\mathrm{D}}{\mathrm{D}t}G.$$

In particular, $\mathcal{F}(\xi,t) \equiv F(\mathbf{x},t)$ satisfies

$$\frac{\partial}{\partial t}\mathcal{F} = 0, \quad \text{hence } \Gamma_s = \left\{\xi \in \mathbb{R}^N \mid \mathcal{F}(\xi) = 0\right\}.$$

One can easily check that the mapping $\xi \mapsto \mathbf{x}$ is ont-to-one from Ω onto $\Omega(t)$, from Γ_s onto $\Gamma_s(t)$ and from Γ_b onto Γ_b. Let $\mathcal{M}$ be a Jacobian matrix

$$\mathcal{M} = \frac{\partial(\mathbf{x})}{\partial(\xi)} = \left(\frac{\partial x_j}{\partial \xi_k}\right)_{j,k=1,2,\dots,N}.$$

Then (5) yields

$$\frac{\partial}{\partial t}\mathcal{M} = \frac{\partial(\mathbf{v})}{\partial(\mathbf{x})}\mathcal{M},$$

from which

(6) $$|\mathcal{M}| \equiv \det\mathcal{M} = 1 \quad \text{(Liouville's theorem)}.$$

Noting that

$$\nabla_x = (\mathcal{M}^*)^{-1}\nabla_\xi \equiv \nabla_u$$

with $\mathcal{M}^*$ being the transposed matrix of $\mathcal{M}$, we transform problem (1)-(4) into the following problem in the fixed domain, which is denoted by Problem A.

Problem A. Find $(\mathbf{u}(\xi.t), p(\xi,t))$ satisfying

$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t} + \dfrac{1}{\varrho}\nabla_{\mathbf{u}} p - \mathbf{f} = 0 \quad \text{in } \Omega,\ t > 0, \\ \nabla_{\mathbf{u}} \cdot \mathbf{u} = 0 \quad \text{in } \Omega,\ t > 0, \\ p = p_e + 2\sigma H \quad \text{on } \Gamma_s,\ t > 0, \\ (\mathbf{v} - \mathbf{v}_b)\cdot \mathbf{n}_b = 0 \quad \text{on } \Gamma_b,\ t > 0, \\ \mathbf{u}|_{t=0} = \mathbf{v}_0(\xi) \quad \text{on } \Omega \quad (\mathrm{div}\mathbf{v}_0 = 0). \end{cases}$$

When $\mathbf{f} = \nabla_x h$ and $p_e = $ constant, one can deduce from Problem A to the following problem.

ProblemA$'$. Find $(\mathbf{u}(\xi.t), q(\xi,t))$ satisfying

$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t} + \dfrac{1}{\varrho}\nabla_{\mathbf{u}} q = 0 \quad \text{in } \Omega,\ t > 0, \\ \nabla_{\mathbf{u}} \cdot \mathbf{u} = 0 \quad \text{in } \Omega,\ t > 0, \\ q = -\varrho h + 2\sigma H \quad \text{on } \Gamma_s,\ t > 0, \\ (\mathbf{v} - \mathbf{v}_b)\cdot \mathbf{n}_b = 0 \quad \text{on } \Gamma_b,\ t > 0, \\ \mathbf{u}|_{t=0} = \mathbf{v}_0(\xi) \quad \text{on } \Omega \quad (\mathrm{div}\mathbf{v}_0 = 0), \end{cases}$$

where $q = p - p_e - \varrho h$.

For Problem A' in case [I] we have the following works. [?], [?], [38], [64], [65].

For Problem A' in case [I] with rotation free case we have the following works. [52], [53], [54], [55], [?], [?], [?], [94], [95]. We give another formulation.

Since operating the material derivative to equation (5) implies

$$\mathbf{x}_{tt} = \frac{\mathrm{D}}{\mathrm{D}t}\mathbf{v},$$

one can derive from (1)

$$\mathcal{M}^* (\mathbf{x}_{tt} - \mathbf{f}) + \frac{1}{\varrho}\nabla_\xi p = 0.$$

Equation (6) is indeed equivalent to equation $(1)^2$, however it is not a divergence form. Following Ovsjannikov, we derive an equivalent divergence form.

For any $\mathcal{A}_j(\mathbf{x}(\xi))$ it holds that

$$\frac{\partial}{\partial \xi_k}\mathcal{A}_j = \mathbf{x}_{\xi_k} \cdot \nabla_x \mathcal{A}_j = \mathrm{div}_x (\mathbf{x}_{\xi_k}\mathcal{A}_j) - \mathcal{A}_j \mathrm{div}_x \mathbf{x}_{\xi_k},$$

and for any Jacobian matrix $\mathcal{A}$

$$\mathrm{div}_x \mathbf{x}_{\xi_k} = \frac{1}{|\mathcal{M}|}\frac{\partial}{\partial \xi_k}|\mathcal{M}|.$$

If $|\mathcal{M}|$ is constant, then it holds that for any $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3)$

$$\mathrm{div}_\xi \mathcal{A} = \mathrm{div}_x (\mathcal{M}\mathcal{A})$$

In our case $|\mathcal{M}| = 1$. Thus $\mathcal{M}\mathcal{A} = \mathbf{x}_t = \mathbf{v}$ and $(1.1)^2$ yield

$$\mathrm{div}_\xi \left(\mathcal{M}^{-1}\mathbf{x}_t\right) = 0.$$

Problem B. To find $(\mathbf{x}(\xi, t), p(\xi, t))$ satisfying

$$(7) \qquad \begin{cases} \mathcal{M}^* (\mathbf{x}_{tt} - \mathbf{f}) + \dfrac{1}{\varrho}\nabla_\xi p = 0 \quad \text{in } \Omega, \ t > 0, \\ \mathrm{div}_\xi (\mathcal{M}^{-1}\mathbf{x}_t) = 0 \text{ or } (1.6) \quad \text{in } \Omega, \ t > 0, \\ p = p_e + 2\sigma H \quad \text{on } \Gamma_s, \ t > 0, \\ (\mathbf{x}_t - \mathbf{v}_b) \cdot \mathbf{n}_b = 0 \quad \text{on } \Gamma_b, \ t > 0, \\ (\mathbf{x}, \mathbf{x}_t)|_{t=0} = (\xi, \mathbf{v}_0(\xi)) \quad \text{on } \Omega \quad (\mathrm{div}\mathbf{v}_0 = 0). \end{cases}$$

Following Weber (1868) (see, for example [81]), we proceed further to deduce the equivalent problem to (4.1) when $\mathbf{f} = \nabla_x h$. Since

$$\mathcal{M}^*\mathbf{x}_{tt} = \frac{\partial}{\partial t}(\mathcal{M}^*\mathbf{x}_t) - \mathcal{M}_t^*\mathbf{x}_t = \frac{\partial}{\partial t}(\mathcal{M}^*\mathbf{x}_t) - \frac{1}{2}\nabla_\xi|\mathbf{x}_t|^2,$$

$(1.8)^1$ is equivalent to

$$\frac{\partial}{\partial t}(\mathcal{M}^*\mathbf{x}_t) + \nabla_\xi\left(p - h - \frac{1}{2}|\mathbf{x}_t|^2\right) = 0. \tag{8}$$

Applying the rotation operator to (8) leads to

$$\frac{\partial}{\partial t}(\mathrm{rot}_\xi \mathcal{M}^*\mathbf{x}_t) = 0,$$

hence

$$\mathrm{rot}_\xi \mathcal{M}^*\mathbf{x}_t = \mathrm{rot}_\xi \mathbf{v}_0.$$

Then one can see that there exists φ such that

$$\mathcal{M}^*\mathbf{x}_t = \nabla_\xi\varphi + \mathbf{v}_0.$$

Note that this φ is generally multi-valued and single-valued if Ω is simply connected. Substituting this into (8) and integration with respect to t imply

$$\varphi_t + p = h + \frac{1}{2}|\mathbf{x}_t|^2 + \chi(t) \quad \text{for } \forall\chi(t).$$

In the following we set $\chi(t) \equiv 0$. ¿From $(7)^2$ it follows

$$\mathrm{div}_\xi\left(\mathcal{M}^{-1}\mathcal{M}^{*-1}(\nabla_\xi\varphi + \mathbf{v}_0)\right) = 0.$$

Finally we arrive at the equivalent problem to **Problem B**:
Problem B$'$. To find $(\mathbf{x}(\xi,t), \varphi(\xi,t))$ satisfying

$$\begin{cases} \mathcal{M}^*\mathbf{x}_t = \nabla_\xi\varphi + \mathbf{v}_0 \quad \text{in } \Omega,\ t > 0, \\ \mathrm{div}_\xi\left(\mathcal{M}^{-1}\mathcal{M}^{*-1}(\nabla_\xi\varphi + \mathbf{v}_0)\right) = 0 \quad \text{in } \Omega,\ t > 0, \\ \varphi_t = h + \frac{1}{2}\left|\mathcal{M}^{*-1}(\nabla_\xi\varphi + \mathbf{v}_0)\right|^2 - p_e - 2\sigma H \quad \text{on } \Gamma_s,\ t > 0, \\ \left(\mathcal{M}^{*-1}(\nabla_\xi\varphi + \mathbf{v}_0) - \mathbf{v}_b\right)\cdot\mathbf{n}_b = 0 \quad \text{on } \Gamma_b,\ t > 0, \\ (\mathbf{x},\varphi)|_{t=0} = (\xi, 0) \quad \text{on } \Omega. \end{cases} \tag{9}$$

We remark a little bit further. Let $\omega \equiv \mathrm{rot}_x\mathbf{v}$. Then $(1.1)^1$ yields

$$\frac{\mathrm{D}}{\mathrm{D}t}\omega - \omega\cdot\nabla_x\mathbf{v} = \mathrm{rot}_x\mathbf{f}.$$

When $\mathbf{f} = \nabla_x h$, this equation becomes

$$\frac{\mathrm{D}}{\mathrm{D}t}\omega - \omega\cdot\nabla_x\mathbf{v} = 0. \tag{10}$$

Therefore

$$\omega = \mathcal{M}\mathrm{rot}_\xi\mathbf{v}_0$$

follows, which is known as Lagrange-Cauchy theorem (cf. [81]). Then it is easily seen that $\mathrm{rot}_x \mathbf{f} = 0$, $\mathrm{rot}_x \mathbf{v}_0 = 0$ if and only if $\omega \equiv 0$.

In the case of rotation free there exist a velocity potential Φ such that $\mathbf{v} = \nabla_x \Phi$. Then $\mathcal{M}^* \mathbf{x}_t = \nabla_\xi \Phi$, which implies $\mathrm{rot}_\xi(\mathcal{M}^* \mathbf{x}_t) = 0$. This means that $\mathcal{M}^* \mathbf{x}_t$ is a potential if and only if $\mathcal{M}^* \mathcal{M}_t$ is symmetric, i.e., $\mathcal{M}^* \mathcal{M}_t = \mathcal{M}_t^* \mathcal{M}$. For a potential flow it follows from (8) that

$$\Phi_t + p = h + \frac{1}{2}|\mathbf{x}_t|^2.$$

Then one can see that Φ is connected with φ appeared in the Weber transformation:

$$\Phi = \varphi + \Phi_0, \quad \Phi_0 = \Phi|_{t=0}.$$

In the two-dimensional case, since $\mathbf{v} = (v_1, v_2, 0)(x, x_2)$, $\mathcal{M}$ and $\omega_3 \equiv \omega$ become

$$\mathcal{M} = \begin{pmatrix} x_{1,\xi_1} & x_{1,\xi_2} & 0 \\ x_{2,\xi_1} & x_{2,\xi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

and

$$\omega = \frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2}.$$

For a potential flow (4.4) becomes

$$\omega = \omega_0(\xi_1, \xi_2) = \frac{\partial v_{0,2}}{\partial \xi_1} - \frac{\partial v_{0,1}}{\partial \xi_2}.$$

¿From the equation

$$\omega = \mathrm{rot}_x = \mathcal{M} \mathrm{rot}_\xi \mathcal{M}^* \mathbf{x}_t$$

it follows that

$$(11)^1 \qquad x_{1,\xi_2} x_{1,\xi_1 t} - x_{1,\xi_1} x_{1,\xi_2 t} + x_{2,\xi_2} x_{2,\xi_1 t} - x_{2,\xi_1} x_{2,\xi_2 t} = \omega_0(\xi_1, \xi_2).$$

(6) implies

$$(11)^2 \qquad x_{1,\xi_1} x_{2,\xi_2} - x_{1,\xi_2} x_{2,\xi_1} = 1.$$

$(7)^3$ becomes

$$\tau \cdot \nabla_\xi p = \tau \cdot \nabla_\xi p_e + 2\sigma \tau \cdot \nabla_\xi H \quad \text{on } \Gamma_s,$$

where $\tau = (\tau_1, \tau_2)$ is a tangential vector to Γ_s. This, together with $(7)^1$, yields

$$(11)^3 \qquad [x_{1,\xi_1}(x_{1,tt} - f_1) + x_{2,\xi_1}(x_{2,tt} - f_2)]\tau_1 + [x_{1,\xi_2}(x_{1,tt} - f_1) + x_{2,\xi_2}(x_{2,tt} - f_2)]\tau_2$$

$$= -\frac{1}{\varrho}\tau \cdot \nabla_\xi p_e - \frac{2\sigma}{\varrho}\tau \cdot \nabla_\xi H.$$

Summing up, the two-dimensional problem for a potential flow is to find $\mathbf{x}$ satisfying

$$\begin{cases} (11)^1, (11)^2 \quad \text{in } \Omega, \ t > 0, \\ (11)^3 \quad \text{on } \Gamma_s, \ t > 0, \\ (7)^4 \quad \text{on } \Gamma_b, \ t > 0, \\ (7)^5 \quad \text{in } \Omega \end{cases}$$

under the conditions $\mathrm{div}_\xi \mathbf{v}_0 = 0, \mathrm{rot}_\xi \mathbf{v}_0 = \omega_0$.

For Problem B' in the three dimensional and the rotation free case Bimenov proved the existence result by virtue of Nash-Moser inplicit theorms [17], [18]. Using the same way, we shall be able to prove the well-posedness for the vortical case.

And Andreev [8], [9] discussed the discussed th stability for Problem B. Following his method, we shall be able to construct the solution around Gerstner's trocoidal solution.

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